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Quadrature-based diffusive representation of the fractional derivative with applications in aeroacoustics and eigenvalue methods for stability

The Riemann-Liouville fractional derivative can be recast into an observer of an infinite-dimensional state φ through its so-called diffusive representation

$$\mathrm{d}^{1/2} f(t) = \int_0^\infty \mu(\xi)\partial_t\varphi(t,\xi)\,\mathrm{d}\xi, \text{ with } \partial_t\varphi(t,\xi) = -\xi\varphi(t,\xi) + f(t),$$

$\varphi(0,\xi) = 0$, and $\mu(\xi) = {}^1/_{\pi\sqrt{\xi}}$. This purely time-local representation of the hereditary operator $\mathrm{d}^{1/2}$ is computationally relevant insofar as it can be conveniently and accurately discretized. A discretization method, which involves a linear least-squares optimization, has proven suitable for time-domain simulations,[1] but it requires an a priori choice of the N_ξ discrete poles ξ_i and leads to spectral pollution, which prevents its use in eigenvalue problems that arise in fractional ordinary (FDE) and partial differential equations (FPDE).

This talk introduces a quadrature-based discretization method whose sole parameter is N_ξ and illustrates its properties on two applications. It is first applied to the stability study of a fractional delay differential equation, by computing the spectrum of the associated infinitesimal generator.[2] It yields accurate and convergent spectra, by contrast with the optimization-based method. The second application is a bidimensional wave propagation problem, namely the linearized Euler equations with a fractional impedance boundary condition.[3] Perspectives include the application to fractional PDEs and the extension to other hereditary operators of diffusive type.

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[1] Lombard, B. and Matignon, D. “Diffusive Approximation of a Time-Fractional Burger’s Equation in Nonlinear Acoustics”. *SIAM Journal on Applied Mathematics* 76.5 (2016), pp. 1765–1791. DOI: 10.1137/16M1062491.

[2] Monteghetti, F., Haine, G., and Matignon, D. “Stability of Linear Fractional Differential Equations with Delays: a coupled Parabolic-Hyperbolic PDEs formulation”. *20th World Congress of the International Federation of Automatic Control (IFAC)*. (Toulouse, France). July 9–14, 2017. DOI: 10.1016/j.ifacol.2017.08.1966.

[3] Monteghetti, F., Matignon, D., and Piot, E. “Energy analysis and discretization of nonlinear impedance boundary conditions for the time-domain linearized Euler equations”. (Submitted).